Technion - Israel Institute of Technology
Department of Computer Science

Analysis of non-rigid surfaces 236611

The course deals with modern methods of analysis of non-rigid surfaces, an important emerging field bringing together different disciplines of mathematics and computer science such as differential and metric geometry, numerical analysis, optimization, computer graphics, machine learning, computer vision and computational geometry. The course objective is to give theoretical and numerical tools for the analysis and comparison of surfaces from the perspective of the recent advances in the field. The first part of the course will include a brief introduction into topology, metric and Riemannian geometry as well as numerical geometry, numerical analysis and state-of-the-art tools in numerical optimization. The second part is dedicated to the representation of intrinsic geometry of surfaces. We will discuss the notion of isometric embedding, discern between local and global isometries and study different numerical methods for analysis and comparison of non-rigid surfaces. Two major emphases are spectral embeddings and multidimensional scaling. The last part of the course is dedicated to applications. We will see how many important problems can be addressed within the framework of non-rigid surface matching. We will demonstrate the 3D face recognition system based on intrinsic geometric representation of faces.

Teachers

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Approximate Weekly Schedule

Week 1: overview of applications, basics of topology and metric geometry.
Week 2: introduction to Riemannian geometry, intrinsic vs. extrinsic geometry, geodesics.
Week 3: basic terms in numerical geometry, surface representation, sampling, triangulation, polyhedral meshes, fast marching methods.
Week 4: basic terms in numerical optimization, gradient descent methods, Newton and quasi-Newton methods.
Week 5: local vs. global optimization, non-convex optimization, majorization, multi-resolution and multi-grid methods.
Week 6: introduction to isometry invariant surface matching, iterative closest point algorithms.
Week 7: the Schwartz and Elad-Kimmel approach, partial surface matching.
Week 8: multidimensional scaling, LS-MDS, SMACOF, multi-grid MDS.
Week 9: non-Euclidean embedding, spherical and hyperbolic MDS, generalized MDS.
Week 10: Gromov-Hausdorff distance, Memoli-Sapiro and Bronstein-Bronstein-Kimmel approaches.
Week 11: spectral embeddings, Isomap, locally linear embedding (LLE), Hessian LLE, SDP embedding, Elad-Kimmel canonical forms, Laplace-Beltrami operator and its spectrum, classical scaling.
Week 12: applications: the isometric model of facial expressions, expression-invariant face recognition, texture mapping, virtual makeup.
Week 13: applications: geometric-variational models, isometric priors in variational models.
Week 14: applications, as time permits.

Pre-requisites

Good knowledge of math, including differential and integral calculus (104011, 104004 or 104282) and linear algebra (104006 or 104167). Advantage for students who studied one or more of the following courses:

- Numerical analysis (104283, 234107)
- Multi-grid methods (236790)
- Optimization (046197, 236330)
- Numerical geometry of images (236861)
- Computer graphics and computational geometry (234325, 046345, 236719)
- Riemannian or differential geometry (106350, 104177, 106723)
- Metric geometry or topology (104142, 104144)

Grading Policy (tentative)

25%	Homework
60%	Course project
15%	Final presentation

Recommended Literature

- D. Burago, Iu. D. Burago, S. Ivanov, A Course in Metric Geometry, American Mathematical Society, 2001, ISBN 0821821296.
- M. Gromov, Metric Structures for Riemannian and non-Riemannian Spaces. Birkhaeuser, 1999, ISBN 0817638989.
- D. P. Bertsekas, Nonlinear Programming (2nd edition), Athena Scientific, 1999, ISBN 1886529000.
- P. E. Gill, W. Murray, M. H. Wright, Practical Optimization, Academic Press, 1982, ISBN 0122839528.
- Borg, P. J. F. Groenen, Modern Multidimensional Scaling: Theory and Applications (2nd edition), Springer, 2005, ISBN 0387251502.
- J. O'Rourke, Computational Geometry in C (2nd edition), Cambridge University Press, 2000, ISBN 0521649765.
- W. L. Briggs, V. E. Henson, S. F. McCormick, A Multigrid Tutorial (2nd edition), SIAM, 2000, ISBN 0898714621.
- S. Rusinkiewicz, M. Levoy, "Efficient Variants of the ICP Algorithm", In *Proc. 3D Digital Imaging and Modeling 2001*, IEEE Computer Society Press, 2001, pp. 145–152.
- J. B. Tenenbaum, V. de Silva, J. C. Langford, "A global geometric framework for Nonlinear Dimensionality Reduction", *Science* 290 (5500): 2319-2323, 22 December 2000.
- S. Roweis, L. Saul, "Nonlinear dimensionality reduction by locally linear embedding", *Science* 290 (5500):2323–2326, 22 December 2000.

- M. Belkin, P. Niyogi, “Laplacian Eigenmaps and Spectral Techniques for Embedding and Clustering”, In T. G. Dietterich, S. Becker, and Z. Ghahramani (eds.), Advances in Neural Information Processing Systems 14, MIT Press, 2002.
- G. Zigelman, R. Kimmel, N. Kiryati, “Texture mapping using surface flattening via multi-dimensional scaling” *IEEE Trans. on Visualization and Computer Graphics,* 8(2):198–207, 2002.
- Elad, R. Kimmel, “On bending invariant signatures for surfaces”, *IEEE Trans. on PAMI*, 25(10):1285–1295, 2003.
- D. Donoho, C. Grimes, “Hessian eigenmaps: Locally linear embedding techniques for high-dimensional data”, *PNAS* 100 (10):5591–5596, 13 May 2003.
- Bronstein, M. Bronstein, R. Kimmel, “Three-dimensional face recognition”, *IJCV*, 64(1):5–30, 2005.
- Bronstein, M. Bronstein, R. Kimmel, I. Yavneh, “A multigrid approach to multidimensional scaling”, In Proc. 12th Copper Mountain Conf. on Multigrid Methods, Colorado, 3-8 April, 2005. To appear in *Numerical Linear Algebra with Applications.*
- F. Memoli, G. Sapiro, “A Theoretical and Computational Framework for Isometry Invariant Recognition of Point Cloud Data”, *Foundations of Computational Mathematics*, published online 30 June 2005.
- M. Reuter, F.-E. Wolter, N. Peinecke, “Laplace-Spectra as Fingerprints for Shape Matching”, *Proc. of the ACM Symposium on Solid and Physical Modeling*, June 2005.
- Bronstein, M. Bronstein, R. Kimmel, “Generalized multidimensional scaling: a framework for isometry-invariant partial surface matching”, *PNAS*, 2006. to appear.