**Partial similarity**

Greek mythology

Visual agnosia

The man who mistook his wife for a hat

Recognition by parts

Am I human? I am a centaur. Am I equine?
**Ingredients**

- Part – subset of the shape \( \mathcal{X} \subseteq \mathcal{X}(\mathbf{x}) \)
- Similar
- Dissimilar
- \( d(\mathbf{x}, \mathbf{y}) \) similarity applied to parts

**Significance**

- Significance = measure of the part \( \mu(\mathcal{X}) \)
- Significant
- Insignificant
- Area: size of the part

**Problem:** how to select significant parts?
- Significance of a part is a semantic definition
- Different shapes may have different definition of significance

**Significance**

- Do all parts have the same importance?
- It really depends on the data

**Statistical significance**

- Frequent = important
- Rare = discriminative
- syzygy in astronomy means alignment of three bodies of the solar system along a straight or nearly straight line. A planet is in syzygy with the earth and sun when it is in opposition or conjunction, the moon is in syzygy with the earth and sun when it is new or full.
Statistical significance

Numerical example

Multicriterion optimization
- Simultaneously minimize dissimilarity and insignificance over all the possible pairs of parts
  \[ \min_{x \in X, \hat{y} \in Y} \{(x', \hat{y}'), \ldots, \mu(x) - \mu(y')\} \]
- This type of problems is called multicriterion optimization
- Vector objective function
- How to solve a multicriterion optimization problem?

Pareto optimality
A solution is said to be a global optimum of an optimization problem
\[ \min_{x \in X} f(x) \]
if there is no other \( z \in X \) such that
\[ f(z) < f(x) \]
A solution is said to be a Pareto optimum of a multicriterion optimization problem
\[ \min_{x \in X} (f_1(x), \ldots, f_n(x)) \]
if there is no other \( z \in X \) such that
\[ f_i(z) < f_i(x) \]
for all \( i \)

Optimum is a solution such that there is no other better solution
In multicriterion case, better = all the criteria are better

Set-valued partial similarity
- The entire Pareto frontier is a set-valued distance
  \[ d_\mathcal{P}(X, Y) = \min_{x, \hat{y} \in X, Y} \{(x', \hat{y}'), \ldots, \mathcal{S}(x', \hat{y}') \} \]
- The dissimilarity at \( \mathbf{s} = \mathbf{0} \) coincides with full similarity \( \mathcal{S}(X, Y) \)
Scalar- vs. set-valued similarity

Order relations
- There is no total order relation between vectors
- As a result, two Pareto frontiers can be non-commeasurable

Scalar-valued partial similarity
- Fix a value of dissimilarity
- Distance from utopia point
- Area under the curve

Characteristic functions
- Problem: optimization over all possible parts $X' \subseteq X$, $Y' \subseteq Y$
- A part is a subset of the shape
- Can be represented using a characteristic function
- Still a problem: optimization over binary-valued variables (combinatorial optimization)
where is a threshold on normals

A. Bronstein, M. Bronstein, A. Bronstein, R. Kimmel, IJCV 2008; A. Bronstein, M. Bronstein, NOSEG 2008

Fréchet's partial similarity

A. Bronstein, M. Bronstein, A. Bronstein, R. Kimmel, IJCV 2008; A. Bronstein, M. Bronstein, NOSEG 2008

Example of intrinsic partial similarity

A. Bronstein, M. Bronstein, A. Bronstein, R. Kimmel, IJCV 2008; A. Bronstein, M. Bronstein, NOSEG 2008

Extra intrinsic partial similarity

How to make ICP compare partially similar shapes?

Introduce weights into the shape-to-shape distance to reject points with bad correspondence

Possible selection of weights:

where is a threshold on normals

and is a threshold on distance

A. Bronstein, M. Bronstein, A. Bronstein, R. Kimmel, IJCV 2008; A. Bronstein, M. Bronstein, NOSEG 2008
Numerical geometry of non-rigid shapes  Partial similarity

Extrinsic partial similarity

Problem: there is no explicit influence of the rejection thresholds and the size of the resulting parts.

Pareto framework allows to control the size of the selected parts!

Not only size matters

What is better?...

...or one large part?

Many small parts...

A. Bronstein, M. Bronstein, A. Bruckstein, R. Kimmel, IJCV 2008; A. Bronstein, M. Bronstein, NORDIA 2008
Numerical geometry of non-rigid shapes Partial similarity

Regularity

Irregular = long boundary
Regular = short boundary

Shape factor (circularity)

\[ r(X) = \frac{4\pi \int_{X} dA}{\int_{X} \left| \partial X \right|^2} \]

Mumford-Shah functional

Salvation comes from image segmentation

\[ \min_{\varphi, \rho} \int_{\Omega} |\nabla u|^2 + \lambda (\varphi - \rho) \, d\Omega \]

replace \( \Omega \) by a membership function \( u : \Omega \to [0, 1] \)

\[ \min_{\varphi, \rho} \int_{\Omega} |\nabla u|^2 + \int_{\Omega} (1 - u) \, d\Omega \]

The two problems are equivalent

Numerical example

ICP example