Problems

Shortest Paths, Fast Marching, Rigid Shapes

January 31, 2009

1. Fast marching methods produce a first order-accurate approximation of the distance map on a mesh, which in itself is a first-order approximation of the underlying smooth surface. However, there exist algorithms like the Mount-Mitchell-Papadimitriou algorithm, that compute *exact* geodesic distances on polyhedra. Demonstrate the advantage of these algorithms by showing on a simple example that geodesic distances on polyhedra are a *second order* approximation of the geodesic distances on the underlying smooth surface.

2. A smooth planar curve $\Gamma \subset \mathbb{R}^2$ defines the distance map

   $$d(x, \Gamma) = \min_{y \in \Gamma} \|x - y\|_2.$$

   Use Taylor expansion to derive a second-order approximation of $d(x, \Gamma)$. What can be said about its smoothness?

3. Show incongruent planar shapes whose first- and central second-order geometric moments are identical.

4. Given two sets of corresponding points $\{x_i\}$ and $\{y_i\}$ and a set of normal vectors $\{n_i\}$ for $i = 1, ..., N$, show a closed-form expression for their best alignment in the sense of the point-to-plane distance,

   $$(R^*, t^*) = \arg\min_{R,t} \sum_{i=1}^{N} \langle n_i, R y_i + t - x_i \rangle^2$$

   where R^* is a rotation matrix and t^* is a translation vector.