Problems

Partial Similarity

Numerical geometry of non-rigid shapes

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1. Show that a Salukwadze optimum is also a Pareto optimum.

2. Show that characteristic functions representing crisp parts belong to M_X.

3. Show that the definition

$$\tilde{d}_{\mathrm{GH}}(m_X, m_Y) = \frac{1}{2} \inf_{\substack{\varphi : X \to Y \\ \psi : Y \to X}} \sigma_{\mathrm{GH}}(m_X, m_Y, \varphi, \psi),$$

where

$$\sigma_{\mathrm{GH}}(m_X, m_Y, \varphi, \psi) = \max \left\{ \begin{array}{c} \sup_{x, x' \in X} m_X(x) m_X(x') |d_X(x, x') - d_Y(\varphi(x), \varphi(x'))| \\ \sup_{y, y' \in Y} m_Y(y) m_Y(y') |d_Y(y, y') - d_X(\psi(y), \psi(y'))| \\ \sup_{\substack{x \in X \\ y \in Y}} m_X(x) m_Y(y) |d_X(x, \psi(y)) - d_Y(\varphi(x), y)| \\ D \sup_{x \in X} \left(1 - m_Y(\varphi(x))\right) m_X(x) \\ D \sup_{y \in Y} \left(1 - m_X(\psi(y))\right) m_Y(y) \end{array} \right\},$$

and $D \geq \max\{\mathrm{diam}(X), \mathrm{diam}(Y)\}$ is equivalent to the Gromov-Hausdorff distance for crisp parts.

4. What will be the consequence of choosing large values of D in the computation of the fuzzy Pareto distance? Explain.

5. Show that if $D = \max\{\text{diam}(X), \text{diam}(Y)\}/\theta(1-\theta)$, where $0 < \theta < 1$ is a parameter, the following relation between d_{P} and $\tilde{d}_{\mathrm{P}}$ holds:

$$\tilde{d}_{\mathrm{P}}(X,Y) \leq \left((1-\theta), \theta^{-2}\right) \cdot d_{\mathrm{P}}(X,Y),$$

   where the inequality is understood in the vector sense.

6. In lossy image and video compression, a fundamental problem is the trade-off between the amount of information used to describe the data (rate) and the amount of error introduced by the compression process (distortion). Describe the rate-distortion optimization problem from the perspective of partial similarity. Prove that the Pareto frontier (the rate-distortion curve) is convex.

7. Show a closed form solution for the best rigid motion between the two sets of corresponding points $\{x_i\}_{i=1}^N$ and $\{y_i\}_{i=1}^N$, minimizing the weighted point-to-point distance

$$\sum_{i=1}^{N} w_i \|x_i - (Ry_i + t)\|_2^2,$$

   where w_i are non-negative weights.