Problems

Numerical Geometry

Numerical geometry of non-rigid shapes

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1. Show that if in a compact metric space there exists an $r/3$-covering containing n points, then an r-separated set cannot contain more than n points.

2. Show that in a compact metric space, a maximal r-separated set is an r-covering.

3. Prove that the farthest point strategy produces an r-separated r-covering.

4. Prove that a convex set is homeomorphic to a disk.

5. Prove that centroidal Voronoi tessellation minimizes the variance of the representation error.

6. Show an example of a geodesic triangle, whose circumscribing ball is non-unique due to insufficient sampling density.

7. Show an example when the Delaunay tessellation of a surface does not exist due to insufficient sampling density.

8. Show an example when the Delaunay tessellation is not unique.

9. Show that given a smooth compact surface X embedded into $\mathbb{R}^3$, there exists an open set U_X such that $X \subseteq U_X$, and a continuous map $\xi : U_X \to X$, such that for all $u \in U_X$, the point $\xi(u)$ is the orthogonal projection of u onto X and it is unique.

10. Show that $|\rho(x) - \rho(x')| \leq d_{\mathbb{R}^3}(x, x')$ for all $x, x' \in X$.

11. Show the relation

$$\rho(x) \leq \frac{1}{\max\{\kappa_1(x), \kappa_2(x)\}},$$

between the local feature size ρ and the maximum curvature radius.

12. Prove that the area of a triangle with vertices $x_1, x_2, x_3 \in \mathbb{R}^3$ can be expressed as $\frac{1}{2}\|(x_2 - x_1) \wedge (x_3 - x_1)\|_2$.

13. Validate the Schwarz lantern example by a formal proof.