Problems
Shortest Paths and Fast Marching Methods
Numerical geometry of non-rigid shapes

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1. Show that a geodesic satisfies $\|\dot{\Gamma}\|_2 = 1$.

2. Prove that the intrinsic gradient $\nabla_X d$ can be obtained by projecting the extrinsic gradient $\nabla_x d$ onto the tangent plane.

3. Prove that a distance function obeys the eikonal equation. Show that the characteristics of the eikonal equation are minimal geodesics.

4. Show that the largest solution of the quadratic equation

$$d_3^2 \cdot 1_{2 \times 1}^{\mathrm{T}} Q 1_{2 \times 1} - 2d_3 \cdot 1_{2 \times 1}^{\mathrm{T}} Q d + d^{\mathrm{T}} Q d - 1 \;\; = \;\; 0.$$

   is associated with the normal direction n, which satisfies the consistency condition $V^{\mathrm{T}} n < 0$.

5. In practical implementation, arithmetics has limited precision. Analyze the sensitivity of the fast marching update scheme to additive truncation and round-off noise. Hint: evaluate the derivative of d_3 with respect to d_1 and d_2.

6. In our discussion, the fast marching update scheme was presented in a special system of coordinates, where the triangle was supposed to lie in the plane. Formulate the fast marching update scheme for a triangle in in an arbitrary system of coordinates in $\mathbb{R}^3$.

7. Devise an alternative update scheme for the fast marching algorithm based on a *circular* wavefront approximation. Prove that such a scheme does not always give a consistent update. What can be said about its numerical stability?

8. There exist algorithms, like the Mitchell-Mount-Papadimitriou algorithm, capable of computing the exact geodesic distances on a triangular mesh. Assuming that the mesh is only a first-order approximation of some underlying continuous surface, are such algorithms have better accuracy compared to fast marching that produces a first-order approximation of the distance map?

9. One of the main limitations of the raster scan algorithm is the fact that it works only with a single-chart parametric surface. However, sometimes it is impractical or impossible to represent a surface using a single chart. Extend the raster scan algorithm to the multiple-chart case.