Problems

In the Rigid Kingdom

Numerical geometry of non-rigid shapes

May 9, 2008

1. Show that the rotation matrix aligning the principal directions with the axes is the diagonalizing matrix of Σ.

2. Try to characterize the class of surfaces completely described by a *finite* set of their geometric moments $\{m_{pqr}\}_{p,q,r=0}^{N}$.

3. Derive a consistent way to discretize the geometric moment integral.

4. Discuss the use of the three-dimensional Fourier harmonics as a replacement to the geometric moments. How can the translation invariance of Fourier harmonics be helpful?

5. In reality, finite-precision arithmetics are used to compute the moments. Assume that the coordinates of the surface points are represented with the absolute error of, say, $\epsilon = 10^{-8}$. What will be the relative error of m_{pqr}? How can this complicate the use of geometric moments?

6. (Research question) Suppose the surface is acquired each time from a different known viewing angle, with partial occlusions. Given the signature of the surface moments for each angle, what can be said about the moments of the entire surface? Can it be reconstructed from such partial observations?

7. Derive the distance in equation

$$\begin{aligned} d^2(y, X) \quad &\approx \quad \frac{d}{d-\rho_1} \langle T_1(x^*), y - x^* \rangle^2 + \frac{d}{d-\rho_2} \langle T_2(x^*), y - x^* \rangle^2 \\ &\quad + \langle N(x^*), y - x^* \rangle^2 , \end{aligned}$$

and show that it is a second-order approximation to the true point-to-surface distance.

8. Prove that the squared point-to-surface distance is not $\mathcal{C}^2$ for query points located on the surface's medial axis.

9. Derive the quadratic form parameters Q, b, and c for the second-order point-to-surface distance. Compare them to the point-to-point and point-to-plane distances. What can be said about the convexity of the quadratic form?

10. Derive a closed-form solution for the optimal rigid isometry (R, t) minimizing the ICP objective function with the squared point-to-point distance.