Problems

Multidimensional Scaling
Numerical geometry of non-rigid shapes

May 9, 2008

1. Verify that the L_2-stress function is non-convex.

2. Derive the matrix form of the L_2-stress.

3. Derive the matrix form of the gradient of the L_2-stress.

4. How does the fact that the matrix $B(Z)$ is zero-mean relate to the degrees of freedom in the MDS problem?

5. Prove the majorization inequality for L_2-stress.

6. Derive the Hessian of the L_2-stress.

7. For the case $m = 2$, show a way to invert the Hessian matrix of the L_2-stress using the inversion formula for block matrices.

8. Show that the constrained optimization problem

$$Z^*_\infty = \underset{Z,\tau}{\operatorname{argmin}}\, \tau \quad \text{s.t.} \quad \begin{cases} d_{ij}(Z) - d_X(x_i, x_j) - \tau \leq 0, \\ -d_{ij}(Z) + d_X(x_i, x_j) - \tau \leq 0, \end{cases}$$

   is equivalent to the L_∞-stress minimization optimization problem

$$Z^*_\infty = \underset{Z}{\operatorname{argmin}} \max_{i,j=1,\ldots,N} |d_{ij}(Z) - d_X(x_i, x_j)|.$$

9. Derive the matrix form of the modified stress and show that the addition of the quadratic penalty does not change the solution of the MDS problem.

10. Derive the relation

$$\nabla_{Z_1}\sigma(Z_1; D_1) - T_1 \quad = \quad P_0^1 \nabla_{Z_0}\sigma(Z_0^{(0)}; D_0),$$

using the chain rule.

11. Show that the least-squares solution to the constrained over-determined linear system

$$A_{K+1}\gamma \quad = \quad 0 \quad \text{s.t.} \quad \gamma_0 + \ldots + \gamma_K = 1.$$

can be found by first solving

$$A^{\mathrm{T}}_{(K+1)} A_{(K+1)} \tilde{\gamma} \quad = \quad 1_{(K+1)\times(K+1)},$$

and then setting

$$\gamma = \frac{\tilde{\gamma}}{\tilde{\gamma}_0 + \ldots \tilde{\gamma}_K}$$