Problems

Non-Euclidean Embedding
Numerical geometry of non-rigid shapes

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1. Write the differential arclength in $\mathbb{S}^m$. Given a path, formulate a functional measuring its length and using the Euler-Lagrange equations, show that the geodesics on a sphere are portions of great circles.

2. Derive the gradient for the stress function in problem

$$\min_{u_1,\dots,u_N} \sum_{i>j} |d_X(x_i,x_j) - d_{\mathbb{S}^m}(u_i,u_j)|^p .$$

3. Show that the spherical embedding problem can be formulated as an algebraic problem similar to classical MDS.

4. Derive the projection operator for the general m-dimensional sphere.

5. Show that the isometry group in $\mathbb{S}^m$ is the orthogonal group $O(m+1)$.

6. Express a rigid isometry in $\mathbb{S}^2$ as a transformation in the parameterization domain.

7. (Research question) Derive an ICP-like algorithm for rigid alignment and matching of surfaces in $\mathbb{S}^3$.

8. (Research question) Derive a method for rigid surface matching in $\mathbb{S}^m$ based on moment signatures.

9. Prove that the interpolation $\hat{d}_Y(u,u') = u'^{\mathrm{T}} D_Y(t,t') u$ satisfies the five properties $\hat{d}_Y$ should satisfy. Is $\hat{d}_Y(u,u')$ continuously differentiable everywhere?

10. Derive a different geodesic distance interpolation scheme based on a spherical wavefront model.

11. Prove that the L_2 stress is convex with respect to each u_i.

12. Derive a closed-form expression for the minimizer of the linearly constrained quadratic stress function $\sigma(u_i)$ in

$$u_i^* = \arg\min_{u_i} \sigma(u_i) \quad \text{s.t.} \quad \begin{cases} u_i \geq 0, \\ u_i^1 + u_i^2 + u_i^3 = 1. \end{cases}$$

13. (Research question) Show a way to introduce point or curve constraints into the GMDS problem, forcing all points lying on some $\Gamma_X \subset X$ to be mapped onto some $\Gamma_Y \subset Y$.