Problems

Isometry-Invariant Similarity

Numerical geometry of non-rigid shapes

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1. Prove that the Gromov-Hausdorff distance obeys all the desired properties listed in the beginning of the chapter.

2. Prove the equivalence of the two definitions

$$d_{\mathrm{GH}}(Y,X) \quad = \quad \inf_{\substack{\mathbb{Z} \\ f:X\to\mathbb{Z} \\ g:Y\to\mathbb{Z}}} d_{\mathrm{H},\mathbb{Z}}(f(X),g(Y)),$$

   and

$$d_{\mathrm{GH}}(Y,X) \quad = \quad \frac{1}{2}\inf_{\substack{\varphi:X\to Y \\ \psi:Y\to X}} \max\{\mathrm{dis}\,\varphi, \mathrm{dis}\,\psi, \mathrm{dis}\,(\varphi,\psi)\}.$$

   of the Gromov-Hausdorff distance. Hint: use the correspondences φ and ψ to define a valid metric on $X \sqcup Y$.

3. Show that the Gromov-Hausdorff distance is consistent to sampling.

4. Consider the following distance

$$d(Y,X) \quad = \quad \frac{1}{2}\inf_{\substack{\varphi:X\to Y \\ \psi:Y\to X}} \max\{\mathrm{dis}\,\varphi, \mathrm{dis}\,\psi\},$$

   which resembles d_{GH} without the term $\mathrm{dis}\,(\varphi,\psi)$. What is the relation between the new distance and the Gromov-Hausdorff distance? Does the new distance obey the similarity property?

5. Consider the following distance

$$d(Y, X) = \frac{1}{2} \inf_{\substack{\varphi : X \to Y \\ \psi : Y \to X}} \operatorname{dis}(\varphi, \psi),$$

What is the relation between the new distance and the Gromov-Hausdorff distance? Does the new distance satisfy the similarity property?

6. (Research question) Consider an L_p formulation of the distortion

$$\operatorname{dis} \varphi = \int_{X \times X} |d_X(x, x') - d_Y(\varphi(x), \varphi(x')|^p d\mu \times \mu(x, x'),$$

where μ is some measure on the surface X (for example, the standard *Hausdorff* or *area measure*). Using the new distortion, devise the L_p Gromov-Hausdorff distance and investigate its properties. Does it satisfy axioms (D)? Devise a new notion of surface similarity satisfied by the L_p Gromov-Hausdorff distance.