[122]. Facial expression may result in significant deformations of the three-dimensional structure of the face, and consequently, we have to think of the face as of a non-rigid surface. This links the face recognition problem to the scope of our book.

13.3 Isometric model of facial expressions

Describing expressions as deformations of the facial surface, we can formulate the problem of *expression-invariant face recognition* as the comparison of facial surfaces invariant to such a class of deformations. The question is whether it is actually possible to model facial expressions. Synthesizing the deformations of the face as the result of expressions appears to be too complicated a task: in the animated movie industry, months are spent on rendering realistic faces of three-dimensional characters. Yet, in our problem, we just need to characterize the class of deformations that result from natural expressions. Here, we have a surprisingly simple result: facial expressions can be approximated by isometries!

In order to convince ourselves in this model, we conducted an experiment, in which a set of flexible markers was attached to the face of a person (Eyal Gordon, then the engineer of the Geometric Image Processing Laboratory, volunteered to act as the "guinea pig" in this experiment). The subject was asked to demonstrate different expressions, ranging from mild to extreme (see some examples in Figure 13.6). By tracking the markers, we could measure the geodesic distances between a set of fixed points on his facial surface. It appeared that the geodesic distances remain approximately invariant to facial expressions. On the other hand, the Euclidean counterparts (describing the extrinsic geometry of the face) show a notably higher variance (Figure 13.7). Later, Mpiperis *et al.* [286, 287] repeated our experiments and independently confirmed the isometric model of facial expressions. Another validation was performed by Gupta *et al.* [194].

The isometric model is of course an approximation; deviations from the model may be attributed to the fact that facial tissues may stretch and therefore are not precisely isometric. For example, expressions with open mouth

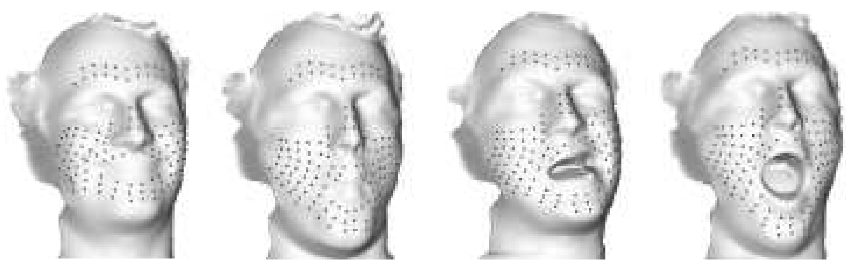

Figure 13.6. Empirical verification of the isometric model of facial expressions. Shown are the points that were tracked.

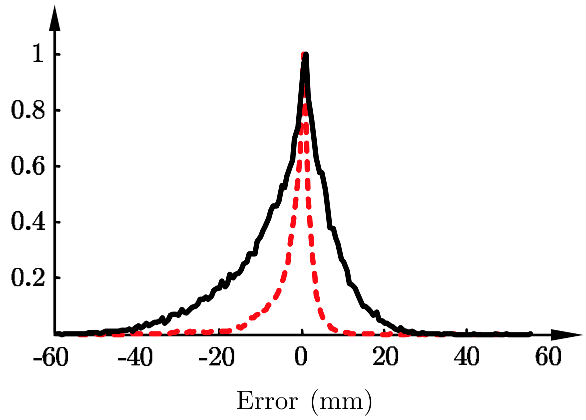

Figure 13.7. Histogram of the geodesic distance deviation from the isometric model (solid); for comparison, a histogram for the Euclidean distances is shown (dashed).

deviate from the isometric model, being topologically different from those with closed mouth. Thus, a geodesic between two points on the upper and the lower lip that crossed the lips when the mouth was closed will circumflex the lip contour when the mouth is open. However, if we introduce a topological constraint into the model, we can easily handle both types of expressions. The easiest way to enforce such a constraint is by cropping out the lips region [73].

13.4 Expression-invariant face recognition

Stated concisely, the isometric model can be formulated as follows: the identity is described by the intrinsic geometry of the face and the expression by the extrinsic one. This means that under the assumption of the isometric model, expression-invariant face recognition is formulated as isometry-invariant comparison of surfaces.

We realized this fact in 2002, thinking of an interesting application for Asi Elad and R.K's canonical forms algorithm. Our first experiments were based on the canonical form distance [63]. A face was first scanned by a coded-light range scanner designed for this purpose, and the input data (in the form of a geometry image) were cropped and sub-sampled to about 2500 points. The matrix of all pair-wise geodesic distances between these points was computed using the fast marching method. Next, the sampled surfaces were embedded into $\mathbb{R}^3$ using the SMACOF algorithm. The obtained canonical form (see examples in Figure 13.8) was aligned by canceling the first-order and the mixed second-order moments, and a signature of moments was computed. The probe signature was then compared with the signatures from the gallery using